Playing card game with finite projective geometry

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CADGME, 2016


DOBBLE
ASMODEE

Natural questions

- How can we construct such cards?
- Does it work with non-8 symbols?
- If yes, does it work with any number of symbols?
- (How many cards are in a deck?)
- How can we realise such cards?

Geometry

Euclid of Alexandria

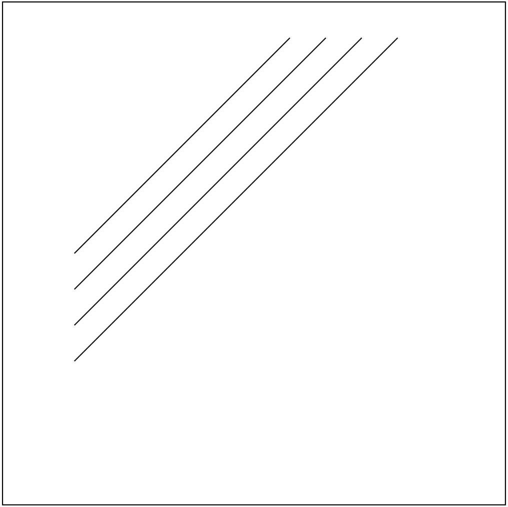
300 BCE

Elements

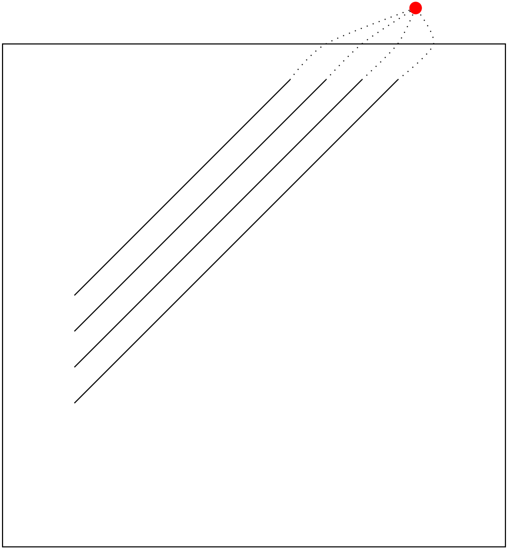
Big problem

Projective plane

Projective plane



Projective plane



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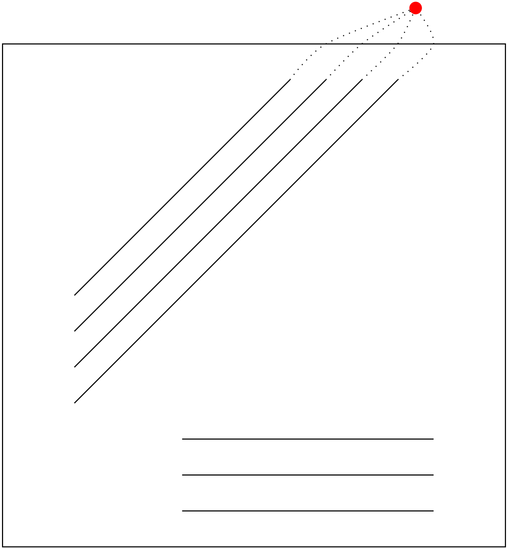

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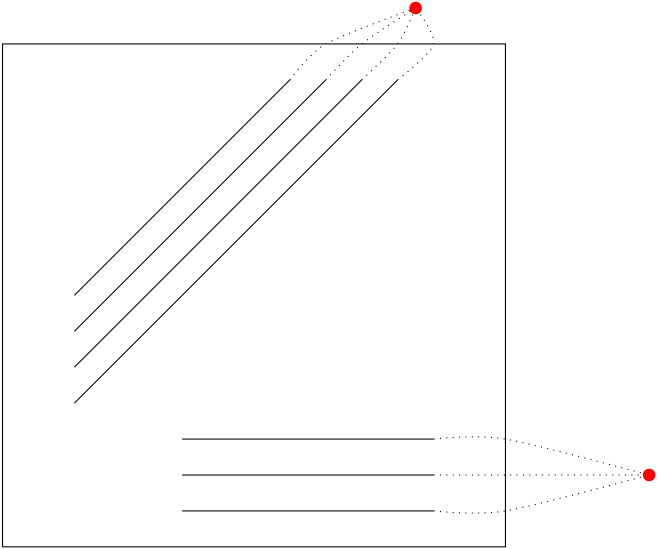

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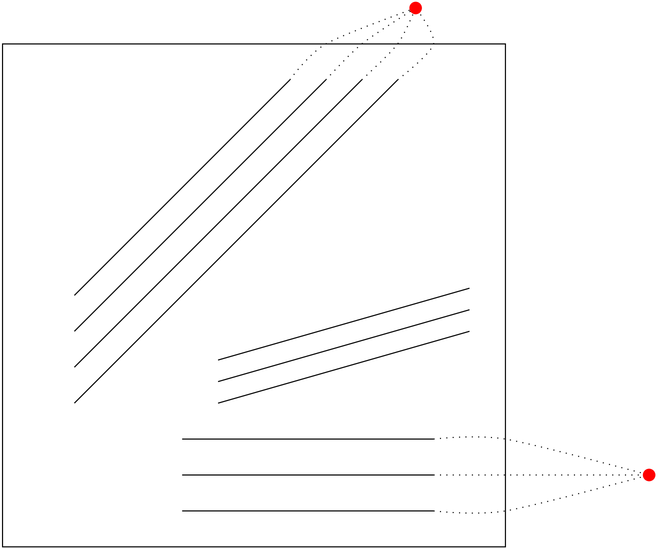

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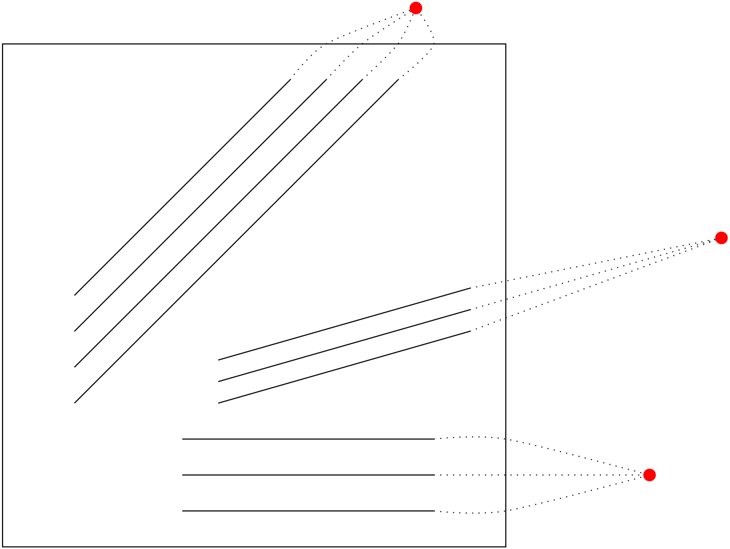

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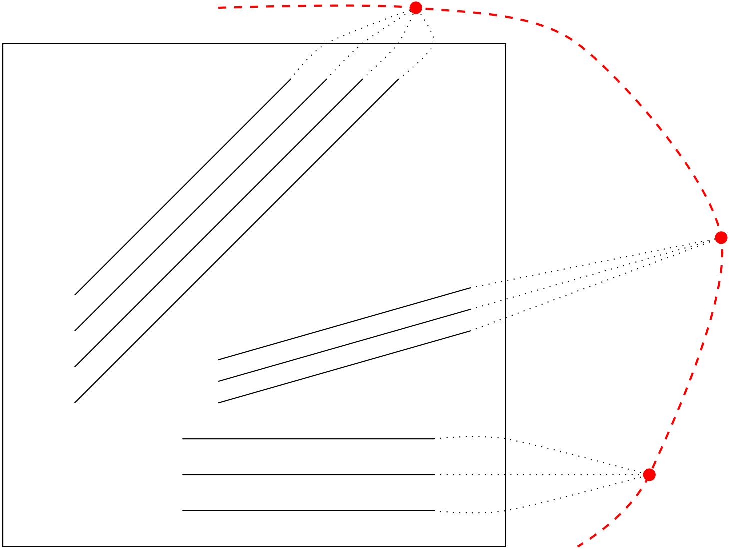


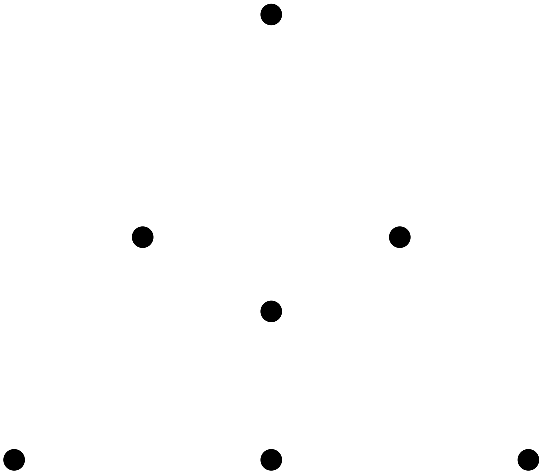
Projective plane

- Given any two distinct points, there is exactly one line incident with both of them.
- There are four points such that no line is incident with more than two of them.
- ~~**Parallel postulate**~~

Instead:

Given any two distinct lines, there is exactly one point incident with both of them.

Fano plane



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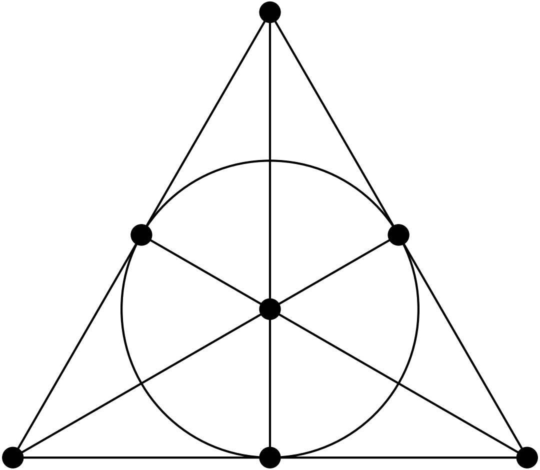

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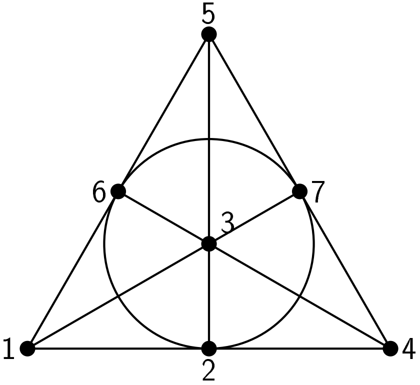
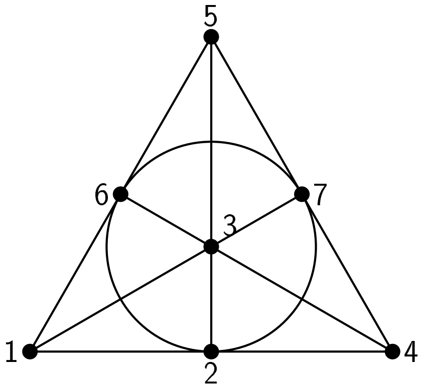


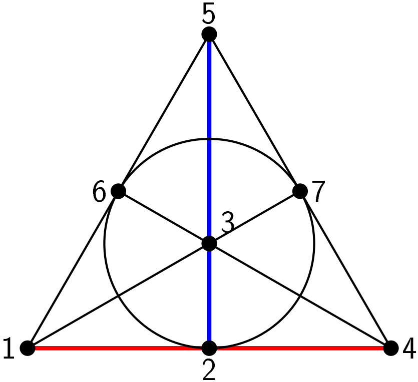
Points: $\{1,2,3,4,5,6,7\}$

Fano plane



Points: $\{1,2,3,4,5,6,7\}$
Lines: $\{\{1,2,4\},\{1,3,7\},\{1,5,6\},\{2,3,5\},\{3,4,6\},\{4,5,7\},\{2,6,7\}\}$

Fano plane



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Dobble revisited: Natural questions

- How can we construct such cards?
- Does it works with non-8 symbols?
- If yes, does it works with any number of symbols?
- (How many cards is in a deck?)
- How can we realise such cards?

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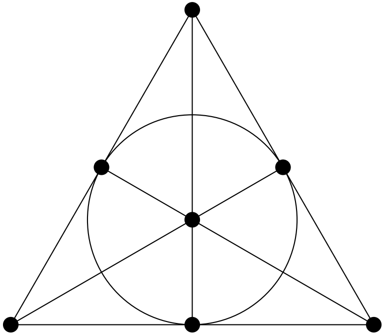
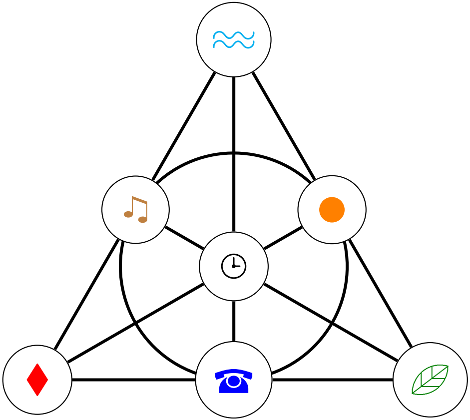


- Point = symbol
- Line = card
- Given any two distinct card, there is exactly one common symbol with both of them.
- Given any two distinct symbols, there is exactly one card with both of them.

Does it works with non-8 symbols?



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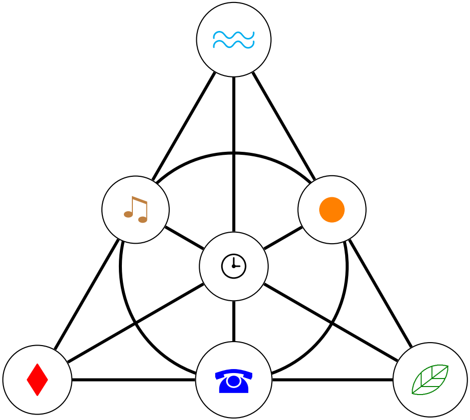

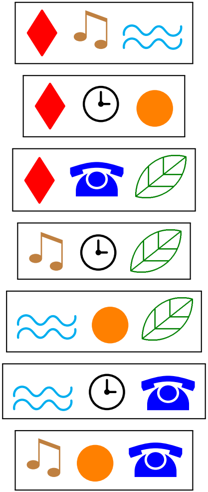

Does it works with any number of symbols?

No.

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No.

Order of the projective plane	# sysmbols per card	
n	$n+1$	
2	3	1
3	4	1
4	5	1
5	6	1
6	7	do not exist
7	8	1
8	9	1
9	10	4
10	11	do not exist

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Conjecture

If n is not prime power then there is no projective plane with order n.

How many cards is in a deck?

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Answer is simple: 55. (We count them.)

How many cards is in a deck?

Theorem

If a projective plane has a line with $n+1$ points then

(1) every line of the plane contains $n+1$ points;

(2) every point of the plane is incident with $n+1$ lines;

(3) the plane has n^2+n+1 points and

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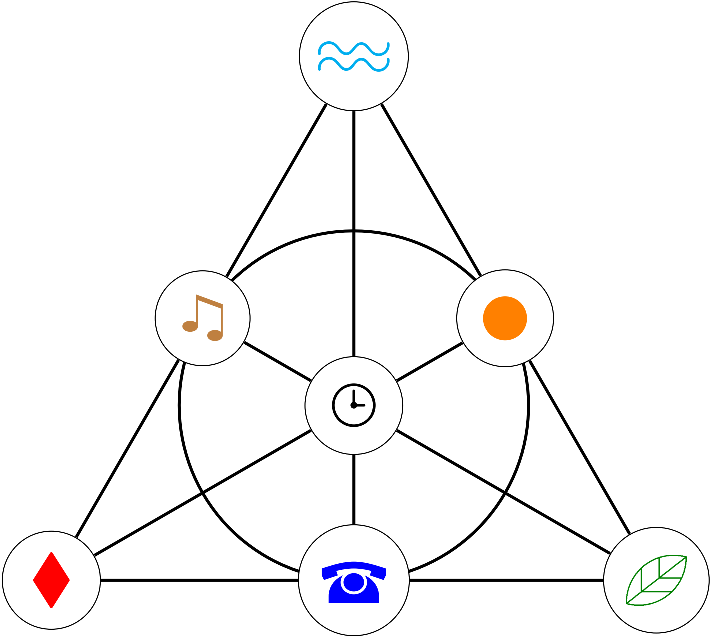
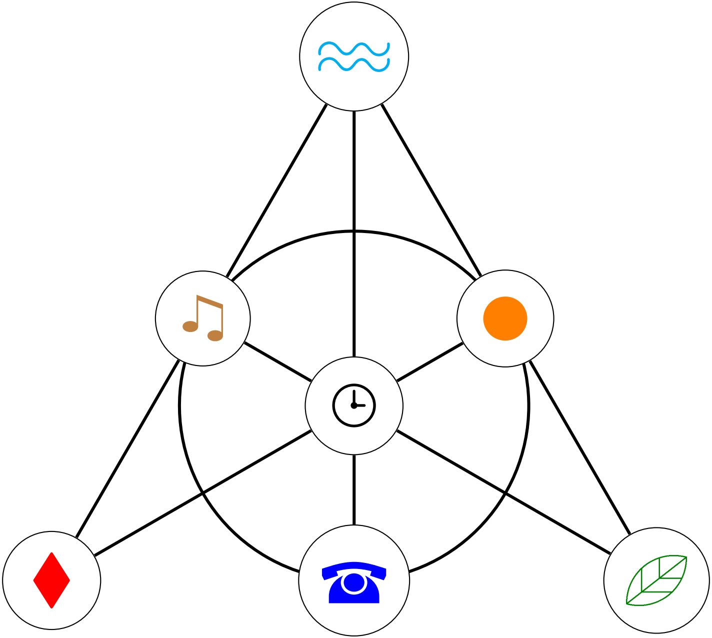
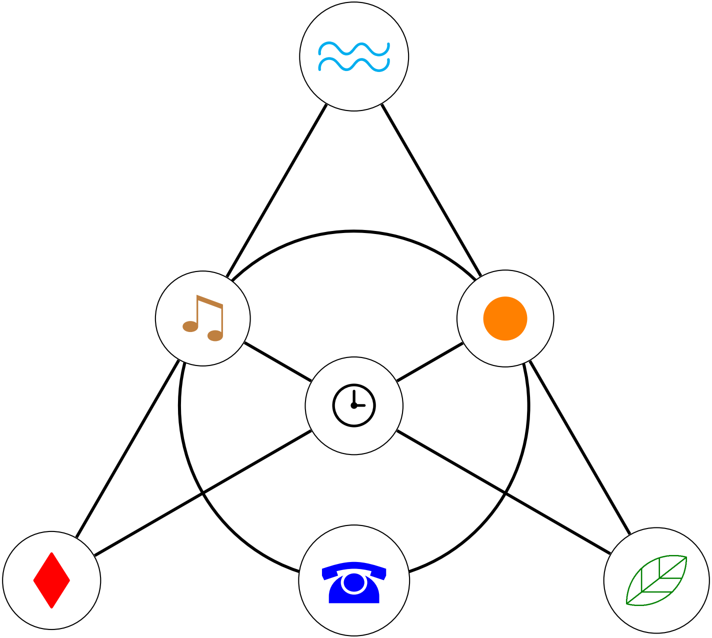
- Where are two missing cards?







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"Answer is simple: 55."

- Where are two missing cards?
- Is this the real model or something else?

How can we realise such cards?

Wolfram Mathematica and GAP demonstrations

Thank you for your attention!